

# Electroweak Sudakov Logarithms and Real Gauge-Boson Radiation in the TeV Region

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## Abstract

Electroweak radiative corrections give rise to large negative, double-logarithmically enhanced corrections in the TeV region. These are partly compensated by real radiation and, moreover, affected by selecting isospin-noninvariant external states. We investigate the impact of real gauge boson radiation more quantitatively by considering different restricted final state configurations. We consider successively a massive abelian gauge theory, a spontaneously broken  $SU(2)$  theory and the electroweak Standard Model. We find that details of the choice of the phase space cuts, in particular whether a fraction of collinear *and* soft radiation is included, have a strong impact on the relative amount of real and virtual corrections.

# 1 Introduction

During the past years electroweak radiative corrections have been evaluated for numerous lepton and hadron collider processes. Despite the relatively small coupling,  $\alpha_W/\pi = \alpha/(\pi \sin^2 \theta_w) \simeq 0.01$ , virtual gauge boson exchange becomes important at high energies, a consequence of the enhancement by large "Sudakov" logarithms [1,2] with the dominant terms proportional to  $\ln^2 s/M_{W,Z}^2$ .

In contrast to QED and QCD, where physical cross sections are obtained by combining virtual and real radiation, events with and without real  $W$ - and  $Z$ -bosons have a distinctly different signature and as such they can in principle be separated in an experimental setup. This observation has led to numerous studies for exclusive reactions [3–26], ranging from purely electroweak four-fermion processes or  $W$ -pair production at electron-positron colliders to the hadronic production of  $Z$ ,  $\gamma$  [27, 28] or  $W$  [29, 30] at large transverse momenta. In some of these cases, in particular for scattering energies in the TeV region, the (negative) one-loop corrections amount to 10% or even up to 30%. This has motivated the investigation of logarithmically enhanced terms of higher orders, either from two-loop effects or in a resummed all-order formulation. To obtain the leading logarithmic (LL) and next-to-leading logarithmic (NLL) terms is straightforward, however, at the same time insufficient for an adequate description e.g. of the dominant two-loop terms. This has motivated studies of higher order contributions and NNLL, partly even NNNLL, results are available for many reactions [9, 10, 22, 23, 25].

The crucial assumption in most of these studies, that events with real gauge boson radiation can be discriminated from the "exclusive" final state, has to be justified by a detailed analysis which obviously depends on the experimental setup. In particular significant differences are expected between electron-positron and hadron colliders, and between leptons or quark and gluon jets in the final state. For "clean" reactions like lepton- or gauge boson-pair production in electron-positron collisions one may anticipate a clear separation, for quark jets in the final state at a hadron collider like the LHC the situation is expected to be more involved.

This has motivated a detailed study of weak boson emission at hadron colliders [31], which demonstrates that, although partial cancellations between virtual and real radiation may occur, the real emission process often only compensates part of the virtual corrections. At first glance one might expect that the combination of virtual and real radiation, the latter completely inclusive, would lead to a complete compensation of the Sudakov logarithms. However, as observed in [32], the preparation of isospin non-invariant external states like electrons or up and down quarks at electron-positron or hadron colliders, respectively, leads to a non-vanishing logarithmically enhanced remainder, a phenomenon called Bloch-Nordsieck violations. These studies were performed in the high-energy limit and real radiation was treated in a completely inclusive manner.

In the present paper we investigate the relative size of virtual versus real radiation, imposing a variety of cuts on the phase space of the emitted gauge boson. These cuts are supposed to represent, in somewhat idealized form, constraints arising from typical detector configurations. As characteristic examples we will consider final states with

soft gauge bosons or, alternatively, with gauge bosons collinear to incoming or outgoing particles. Our considerations will allow to "interpolate" between the completely exclusive and the inclusive treatments. Furthermore, for simplicity, the discussion will be limited to four-fermion processes.

The outline of this paper is as follows. In Section 2 we work out the generic structure of Sudakov logarithms in real emission processes with phase space cuts. While this discussion will be limited to an abelian gauge theory, we investigate the structure and the numerical impact of the Bloch-Nordsieck violations in a spontaneously broken  $SU(2)$  theory in Section 3. The size of the Bloch-Nordsieck violations will be compared to the difference between the fully inclusive result and the one with restricted phase space. In Section 4 our predictions for the Standard Model are presented. As a working example we consider the process  $e^+e^- \rightarrow q\bar{q}$ . We compute next-to-leading order (NLO) electroweak corrections and, in particular, investigate the compensation of the Sudakov suppression from unobservable  $W$ - and  $Z$ -boson radiation. We finally conclude in Section 5.

## 2 Real emission with phase space restrictions

We first pursue the question to which extent the virtual Sudakov corrections are compensated by the real emission process if the latter is subject to certain phase space restrictions. In contrast to the familiar picture from QED or QCD, where an infrared-safe observable necessarily requires inclusion of soft *and* collinear gauge boson emission, it will be instructive for our purposes to also consider scenarios that allow for soft *or* collinear radiation. The physical relevance of the particular phase space cuts will depend on the details of the observable under consideration. We therefore relegate this question to our phenomenological analysis in Section 4 and concentrate for the moment on the generic structure of Sudakov logarithms in real emission processes with phase space restrictions.

It will be convenient for the current discussion to work in a first step within a toy theory that captures the physics of interest while allowing for a compact and transparent presentation. To be specific we consider an abelian gauge theory with explicit mass term  $\frac{1}{2}M^2 A_\mu A^\mu$  that spoils the gauge invariance, but leads, nevertheless, to a consistent renormalizable theory. In the remainder of this section we first address Sudakov logarithms that arise from final state radiation, subsequently we generalize the discussion to the four-fermion process.

### 2.1 Final state radiation

Let us start with an elementary process, namely with the decay of a heavy vector boson (with mass  $\sqrt{s}$ ) into a pair of massless fermions. We assume that this initial vector boson does not couple to our toy theory and that the decay is mediated at Born level by some other vectorlike interaction which we do not specify further. The one-loop virtual corrections are then entirely encoded in the abelian vector form factor (in the timelike region), which has been the central object in the study of electroweak Sudakov logarithms.

In the Sudakov limit  $s \gg M^2$ , the explicit one-loop calculation yields

$$\Gamma^{(V)} \simeq \frac{\alpha}{4\pi} \left\{ -2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + \frac{2\pi^2}{3} - 7 \right\} \Gamma_B, \quad (1)$$

where the fermion wave functions have been renormalized in the on-shell scheme and  $\Gamma_B$  is the Born decay rate. The result reveals a characteristic structure that dominates the decay rate in the high-energy limit. It contains a *Sudakov factor*  $\alpha/(4\pi) [\ln^2 s/M^2 - 3 \ln s/M^2]$  with negative weight (and proportional to the charge squared) for each of the interacting fermions. We will see in the following section that the Sudakov factor is process-independent and contains the full information about collinear logarithms.

We next consider the corresponding real emission process, where the light vector boson is emitted from the final state fermions. Without any restriction on the phase space of the emitted boson, the decay rate becomes in the Sudakov limit

$$\Gamma^{(R)} \simeq \frac{\alpha}{4\pi} \left\{ 2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] - \frac{2\pi^2}{3} + 10 \right\} \Gamma_B. \quad (2)$$

We see that the logarithmic terms cancel in the sum of virtual and real corrections, in accordance with the expectations from the Kinoshita-Lee-Nauenberg (KLN) theorem [33, 34].

Let us now examine how the pattern of these logarithms changes, when we impose different restrictions on the phase space of the emitted boson. We first consider a scenario that allows for soft *and* collinear radiation. To this end we require that the final state fermions are almost back-to-back in the center of mass frame, i.e. we impose a cutoff on the opening angle of the fermion pair,  $\theta_{f\bar{f}} \geq \theta_{f\bar{f}}^c$ , with  $\theta_{f\bar{f}}^c$  close to  $180^\circ$ . In other words we only exclude hard and non-collinear radiation. We now obtain for the restricted real emission process (with  $c_{f\bar{f}}^c \equiv \cos \theta_{f\bar{f}}^c$ )

$$\begin{aligned} \Gamma^{(R)}(\theta_{f\bar{f}}^c) \simeq \frac{\alpha}{4\pi} \left\{ 2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + 4\text{Li}_2 \left( -\frac{1-c_{f\bar{f}}^c}{1+c_{f\bar{f}}^c} \right) \right. \\ \left. + \frac{8(2-c_{f\bar{f}}^c)}{(1-c_{f\bar{f}}^c)^2} \ln \left( \frac{2}{1+c_{f\bar{f}}^c} \right) + \frac{1-5c_{f\bar{f}}^c}{1-c_{f\bar{f}}^c} - \frac{2\pi^2}{3} \right\} \Gamma_B, \end{aligned} \quad (3)$$

which holds for  $s \gg M^2$  and  $1+c_{f\bar{f}}^c \gg M^2/s$ . We see that the given phase space cut does not modify the structure of the mass singularities at all. As the restricted phase space covers all of the singular regions, we again obtain the full Sudakov factors and hence observe a complete cancellation between virtual and real Sudakov logarithms.

It is also interesting to consider a highly restricted phase space in the given scenario, which corresponds to the limit  $\theta_{f\bar{f}}^c \rightarrow 180^\circ$ . We then find

$$\begin{aligned} \Gamma^{(R)}(\theta_{f\bar{f}}^c \rightarrow 180^\circ) \simeq \frac{\alpha}{4\pi} \left\{ 2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] \right. \\ \left. - 2 \left[ \ln^2 \left( \frac{2}{1+c_{f\bar{f}}^c} \right) - 3 \ln \left( \frac{2}{1+c_{f\bar{f}}^c} \right) \right] - \frac{4\pi^2}{3} + 3 \right\} \Gamma_B, \end{aligned} \quad (4)$$

which illustrates how mass singularities are translated into phase space logarithms if only a small fraction of soft and collinear radiation is taken into account. The question of whether or not the Sudakov logarithms numerically dominate the decay rate finally depends on the size of the argument in the phase space logarithms. From our explicit result we are led to expect that the virtual Sudakov logarithms are largely compensated if only a loose cutoff on the opening angle of the fermion pair is applied with  $\theta_{f\bar{f}}^c \lesssim 160^\circ$ .

We next consider a different scenario that allows for collinear radiation only. We now require that the emitted boson is almost parallel to one of the final state fermions, i.e. we impose the constraints  $\theta_{fb} \leq \theta_{Fb}^c$  or  $\theta_{\bar{f}b} \leq \theta_{Fb}^c$  on the angles between the emitted vector boson and the outgoing fermions. Let us now focus for simplicity on the singular part, which is found to be (with  $c_{Fb}^c \equiv \cos \theta_{Fb}^c$ )

$$\Gamma^{(R)}(\theta_{Fb}^c) \simeq \frac{\alpha}{4\pi} \left\{ 2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] - 4 \ln \left( \frac{1 + c_{Fb}^c}{1 - c_{Fb}^c} \right) \ln \frac{s}{M^2} + \dots \right\} \Gamma_B, \quad (5)$$

which holds for  $s \gg M^2$  and  $1 \geq 1 - c_{Fb}^c \gg M^2/s$  (we are actually only interested in the region  $1 \gg 1 - c_{Fb}^c \gg M^2/s$ ). Whereas the double logarithms again cancel between virtual and real corrections, the linear logarithms do not (for  $\theta_{Fb}^c < 90^\circ$ ). The incomplete cancellation reflects the fact that the considered scenario does *not* cover all of the singular regions, it misses in particular soft radiation that escapes the two cones around the final state fermions. We thus expect that the compensation of the virtual Sudakov logarithms is again significant but less effective in this scenario.

For completeness let us also consider a scenario that allows for soft radiation only. We now impose a cutoff on the momentum of the vector boson  $|\vec{k}| \leq k_c$  or, equivalently, on the invariant mass of the fermion pair  $Q^2 \geq Q_c^2$ . As long as we do not cut into the endpoint region, i.e. for  $k_c \gg M$  or  $s - Q_c^2 \gg 2M\sqrt{s}$ , we obtain (with  $z_c \equiv Q_c^2/s$ )

$$\Gamma^{(R)}(z_c) \simeq \frac{\alpha}{4\pi} \left\{ 2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + 2 \left[ 4 \ln(1 - z_c) + 2z_c + z_c^2 \right] \ln \frac{s}{M^2} + \dots \right\} \Gamma_B. \quad (6)$$

We thus find a situation that is conceptually similar to the one before. We again observe an incomplete cancellation of the linear logarithms (for  $z_c > 0$ ) since part of the singular region from (hard-)collinear emission is missed.

## 2.2 Four-fermion process

The preceding example allowed us to study the generic structure of Sudakov logarithms in real emission processes with some exemplary (and idealized) phase space restrictions. Before making any quantitative statements, let us now switch to the four-fermion process which brings in two new aspects. First, we have to deal with initial state radiation and, second, we have to consider the interplay of several phase space restrictions.

The one-loop virtual corrections to the  $s$ -channel four-fermion process  $f'f' \rightarrow f\bar{f}$  amount to the calculation of two form factor type corrections, two box diagrams and the vacuum polarization. As the interference between tree and box diagrams vanishes in our

abelian toy theory, the result takes a particularly simple form

$$\sigma^{(V)} \simeq \frac{\alpha}{4\pi} \left\{ -4 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + \frac{4\pi^2}{3} - 14 - \frac{40}{9} n_f - \frac{16}{9} n_s \right\} \sigma_B, \quad (7)$$

where  $\sigma_B \simeq 4\pi\alpha^2/3s$  is the Born cross section. We thus obtain twice the form factor correction (1) and a contribution from the vacuum polarization<sup>1</sup>.

In the corresponding real emission process the vector boson can be emitted from initial and final state fermions. In contrast to the previous example, we now impose a cutoff on the invariant mass of the fermion pair from the beginning,  $Q^2 \geq Q_c^2 \gg M^2$ , which allows us to circumvent the  $s$ -channel resonant contribution from the initial state radiation (which is of no particular interest for us since we focus on final state configurations that resemble the four-fermion process). Without further restrictions on the emission process, the cross section now becomes in the logarithmic approximation (with  $z_c \equiv Q_c^2/s$ )

$$\begin{aligned} \sigma^{(R)}(z_c) \simeq \frac{\alpha}{4\pi} \left\{ 4 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] \right. \\ \left. + 2 \left[ 1 + 4z_c + z_c^2 - 2 \ln \frac{z_c}{(1-z_c)^4} \right] \ln \frac{s}{M^2} + \dots \right\} \sigma_B, \end{aligned} \quad (8)$$

where we assumed that  $M^2 \ll Q_c^2 \ll s - 2M\sqrt{s}$ . Due to the explicit cutoff  $Q_c^2$ , we thus start with a mismatch between linear virtual and real logarithms from the beginning.

Even if we considered a fully inclusive observable, we actually would not expect the logarithms from initial state radiation to completely cancel the corresponding virtual ones. This may be illustrated with the differential cross section in  $z \equiv Q^2/s$ ,

$$\begin{aligned} \frac{d\sigma^{(R)}}{dz} \simeq \frac{\alpha}{4\pi} \left\{ \frac{1}{z} \left( 4 \frac{1+z^2}{1-z} \ln \frac{(1-z)^2 s}{zM^2} - 8(1-z) \right) \right. \\ \left. + \left( 4 \frac{1+z^2}{1-z} \ln \frac{(1-z)^2 s}{M^2} - 8(1-z) \right) \right\} \sigma_B, \end{aligned} \quad (9)$$

where the first (second) line contains the initial (final) state radiation<sup>2</sup>. Integrating the second line in the kinematic limits  $0 \leq z \leq (1 - M/\sqrt{s})^2$ , we recover the Sudakov logarithms from the inclusive final state radiation in (2). For the initial state radiation, however, we have to proceed differently to single out the logarithms that match the according virtual ones. Applying the usual prescription for plus-distributions, we get

$$\sigma_{\text{initial}}^{(R)} \simeq \frac{\alpha}{4\pi} \left\{ 2 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + 4 \int_0^1 \frac{dz}{z} \left[ \frac{1+z^2}{1-z} \ln \frac{(1-z)^2 s}{M^2} \right]_+ + \dots \right\} \sigma_B, \quad (10)$$

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<sup>1</sup>We assume that the theory contains  $n_f$  massless fermions and  $n_s$  (light) scalar bosons and renormalize the coupling constant in the  $\overline{\text{MS}}$ -scheme. We further set the renormalization scale  $\mu = \sqrt{s}$ .

<sup>2</sup>The interference between initial and final state radiation vanishes in the abelian toy theory in analogy to the cancellation of the box diagrams mentioned above.

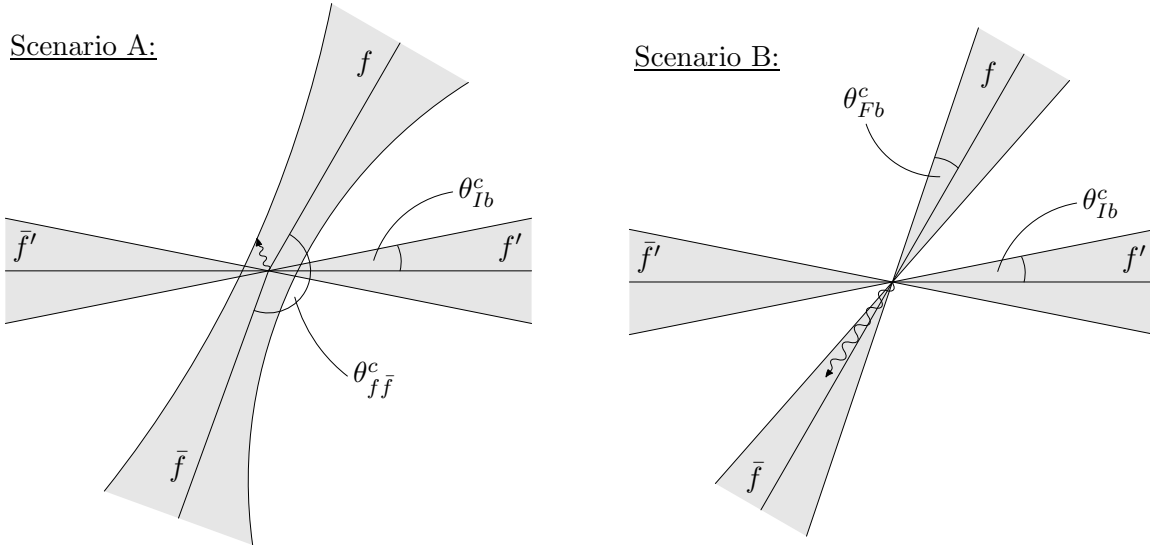


Figure 1: *Different restrictions on the real emission process. The momentum of the undetected gauge boson (wavy line) is assumed to lie within the shaded area. In Scenario A (collinear and soft) we require the final state fermions to be almost back-to-back or the emitted boson to lie within a cone around the initial state fermions. In Scenario B (collinear) the emitted boson has to be within any of the cones around the fermions.*

which illustrates that there is a single collinear logarithm left that does not cancel between virtual and real corrections. The reason for this incomplete cancellation is of course well-known; according to the KLN theorem we would have to account for *incoming* vector bosons to recover complete cancellation. In QCD applications the remnant collinear singularity is usually factorized into process-independent parton distribution functions and similar methods are used in the context of QED. For the weak interactions with a physical gauge boson mass, however, there is no need to factorize this contribution and one is left with a certain mismatch in a fixed-order calculation.

The second new element of the four-fermion process consists in the fact that we want to impose several phase space restrictions at once. In particular we find it convenient to distinguish the following two scenarios, which we will reconsider in our phenomenological analysis in Section 4 (for an illustration of the scenarios cf. Figure 1):

- In Scenario A we combine virtual corrections with real gauge boson radiation, if the final state fermions are almost back-to-back, with an opening angle  $\theta_{f\bar{f}} \geq \theta_{f\bar{f}}^c$ , *or* if the emitted gauge boson is almost collinear to one of the initial state fermions, i.e. if  $\theta_{f'b} \leq \theta_{Ib}^c$  or  $\theta_{\bar{f}'b} \leq \theta_{Ib}^c$ . Applying these phase space restrictions in addition to the  $Q^2$ -cut discussed above, we find the logarithmic terms to be

$$\sigma^{(R)}(z_c, \theta_{Ib}^c, \theta_{f\bar{f}}^c) \simeq \frac{\alpha}{4\pi} \left\{ 4 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + 2 \left[ 1 + 4z_c + z_c^2 - 2 \ln \frac{z_c}{(1-z_c)^4} \right] \ln \frac{s}{M^2} + \dots \right\} \sigma_B. \quad (11)$$

In comparison with (8) we see that the additional phase space cuts  $\theta_{Ib}^c$  and  $\theta_{f\bar{f}}^c$  do not further modify the pattern of the Sudakov logarithms since all of the singular regions (collinear to initial and final state and soft) are covered in this scenario. We therefore expect a strong cancellation of the virtual corrections even for tight cuts on  $\theta_{Ib}^c$  and  $\theta_{f\bar{f}}^c$ .

- In Scenario B we require that the undetected vector boson is almost collinear to one of the initial state fermions, i.e.  $\theta_{f'b} \leq \theta_{Ib}^c$  or  $\theta_{\bar{f}'b} \leq \theta_{Ib}^c$ , *or* to one of the final state fermions,  $\theta_{fb} \leq \theta_{Fb}^c$  or  $\theta_{\bar{f}b} \leq \theta_{Fb}^c$ . As we do not account for soft radiation that escapes the four cones around the fermions, we now expect a somewhat modified logarithmic structure and, consequently, the compensation of the virtual corrections to be less effective. Specifically, we now obtain

$$\begin{aligned} \sigma^{(R)}(z_c, \theta_{Ib}^c, \theta_{Fb}^c) \simeq \frac{\alpha}{4\pi} \left\{ 4 \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] \right. \\ \left. + 2 \left[ 1 + 4z_c + z_c^2 - 2 \ln \frac{z_c}{(1-z_c)^4} - g(\theta_{Ib}^c, \theta_{Fb}^c) \right] \ln \frac{s}{M^2} + \dots \right\} \sigma_B, \end{aligned} \quad (12)$$

with  $(c_{Ib}^c \equiv \cos \theta_{Ib}^c, c_{Fb}^c \equiv \cos \theta_{Fb}^c)$

$$\begin{aligned} g(\theta_{Ib}^c, \theta_{Fb}^c) = \frac{c_{Ib}^c(3 + (c_{Ib}^c)^2)}{2} \ln \left( \frac{1 + c_{Fb}^c}{1 - c_{Fb}^c} \right) + \frac{c_{Fb}^c(3 + (c_{Fb}^c)^2)}{2} \ln \left( \frac{1 + c_{Ib}^c}{1 - c_{Ib}^c} \right) \\ + \frac{3c_{Ib}^c c_{Fb}^c}{2} (2 - (c_{Ib}^c)^2 - (c_{Fb}^c)^2). \end{aligned} \quad (13)$$

In Figure 2 we illustrate these observations quantitatively. First of all we note that the virtual corrections induce a substantial Sudakov suppression in the TeV regime (in the abelian toy theory with Standard Model inspired values  $M = 80$  GeV and  $\alpha = 0.03$ ). Depending on the phase space cuts this suppression is more or less compensated by the real emission process. In the upper plot we illustrate the dependence on the cut on the invariant mass of the fermion pair,  $z_c = Q_c^2/s$ , which is found to have a large impact on the compensation (no angular cut has been applied so far). In the remaining plots we fix  $z_c = 0.5$ , i.e. the middle dashed line from the first plot is the upper solid reference line for the other two plots. We, moreover, impose a rather tight cut on initial state radiation by setting  $\theta_{Ib}^c = 10^\circ$  (the corresponding effect is indicated by the dotted curves). From the middle plot it is evident that the (unobserved) real radiation has a large impact in Scenario A, even when tight phase space cuts as  $\theta_{f\bar{f}}^c = 175^\circ$  are applied. For more moderate cuts as  $\theta_{f\bar{f}}^c = 165^\circ$  the virtual corrections are almost completely compensated in this setup. In Scenario B the compensation is found to be less effective. For moderate cuts as  $\theta_{Fb}^c = 30^\circ$  the virtual corrections are reduced, for instance, from  $-36\%$  to  $-29\%$  at 3 TeV. This comparison illustrates also quantitatively the importance of covering all of the singular phase space regions.



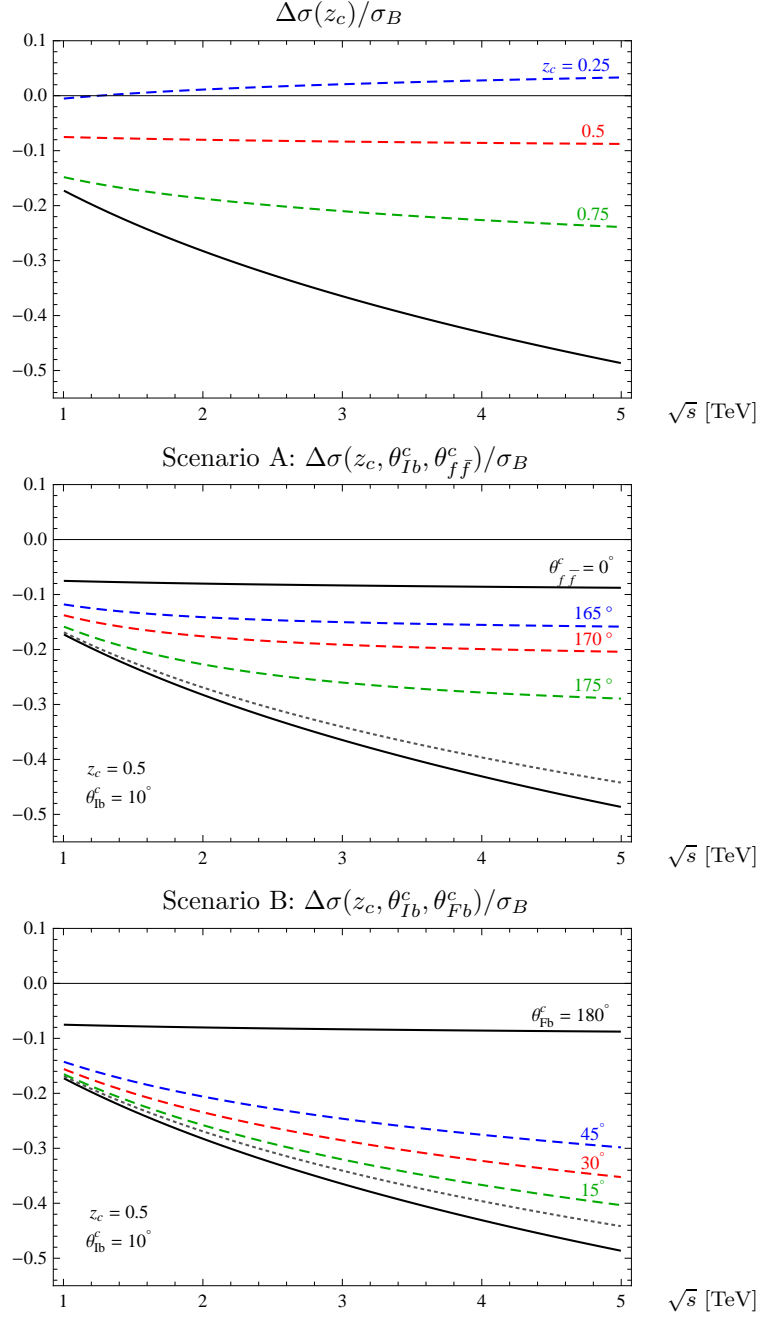


Figure 2: *Relative NLO corrections to the four-fermion process in the abelian toy theory as a function of the center of mass energy  $\sqrt{s}$  in TeV. In each plot the lower solid line represents the virtual correction (with  $M = 80$  GeV,  $\alpha = 0.03$ ,  $n_f = 6$ ,  $n_s = 1$ ) and the dashed lines refer to the sum  $\Delta\sigma = \sigma^{(V)} + \sigma^{(R)}$  with different restrictions on the real emission process. The individual dashed lines (green/red/blue, from bottom to top in each plot) refer to  $z_c = 0.75/0.5/0.25$  and no angular cut (top),  $\theta_{f\bar{f}}^c = 175^\circ/170^\circ/165^\circ$  (middle) and  $\theta_{fb}^c = 15^\circ/30^\circ/45^\circ$  (bottom). In the lower two plots we fixed  $z_c = 0.5$  and  $\theta_{fb}^c = 10^\circ$ . The dotted curves indicate the contribution from initial state radiation (corresponding to  $\theta_{f\bar{f}}^c = 180^\circ$  and  $\theta_{fb}^c = 0^\circ$ , respectively).*

### 3 Bloch-Nordsieck violations

The pattern of Sudakov logarithms is more complicated in non-abelian gauge theories. The non-abelian group structure leads, in particular, even for inclusive observables (with respect to phase space) to a mismatch between virtual and real Sudakov logarithms as long as one does not sum over the non-abelian charges of the external particles. This mismatch, commonly referred to as Bloch-Nordsieck (BN) violations, turns out to be irrelevant in practical QCD applications, since the confinement of the coloured partons into colour-neutral hadrons enforces the summation (or average) over the colour charges. The spontaneous breakdown of the electroweak interactions, however, allows to prepare external states with definite weak isospin. Consequently, even inclusive observables are affected by electroweak Sudakov logarithms [32].

In this section we reconsider the four-fermion process in a spontaneously broken  $SU(2)$  theory to study the structure and the numerical impact of the BN violations. Whereas the gauge bosons  $W^{\pm,3}$  acquire a common mass  $M$  in this non-abelian toy theory, the fermions are again supposed to stay massless and to have a vectorlike coupling to the gauge bosons. In the following we first address the structure of the BN violations on the level of the total cross section, then we switch to a quantitative analysis that accounts for the various phase space restrictions that we introduced in the previous section.

#### 3.1 Structure of Sudakov logarithms

The dynamical origin of Sudakov logarithms is well understood; they are tied to the infrared structure of the theory and arise from collinear or soft radiation of (almost) massless particles. Whereas previous analyses have mainly focused on electroweak Sudakov logarithms from virtual particle exchange (cf. e.g. [4, 9, 10, 24, 25, 30]), electroweak Sudakov logarithms from real emission processes have received less attention so far [32, 35]. Let us therefore recall the origin and the structure of the BN violations in the considered  $SU(2)$  theory in some detail. This will help us later to translate the results to Standard Model processes.

Let us start the discussion with the collinear approximation, which is known to yield an universal radiation factor for each external particle. This can be seen most easily in an axial gauge, where the collinear logarithms stem from self energy insertions into external lines. For the four-fermion process with generic isospin charges,  $f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4$ , the virtual collinear logarithms associated with the outgoing fermion  $f_3$  amount, for instance, to

$$\sigma^{(V, col \ f_3)} \simeq -\frac{\alpha}{4\pi} \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] (T^A T^A)_{f_3 f'} \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f' \bar{f}_4} \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} \sigma_B^0, \quad (14)$$

where  $T^A$  denotes a generator of the  $SU(2)$  group and we made the (real-valued) group structure of the Born amplitude,  $\mathcal{A}_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} = \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} \mathcal{A}_B^0$ , explicit ( $\sigma_B^0 \simeq 4\pi\alpha^2/3s$  is the Born cross section of the abelian theory and a summation over  $A$  and  $f'$  is understood). In the collinear approximation we thus obtain a Sudakov factor with negative weight for each external fermion, which is to be multiplied with a Casimir factor  $(T^A T^A)_{f_3 f'} = C_F \delta_{f_3 f'}$ .

It is convenient to disentangle the contributions from  $W^3$  and  $W^\pm$  exchange and to write the result as

$$\sigma^{(V, col \ f_3)} \simeq -\frac{\alpha}{4\pi} \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] \left( (t_{f_3}^3)^2 + (t^\pm)^2 \right) \sigma_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4}, \quad (15)$$

where  $t_{f_3}^3$  denotes the isospin of the fermion  $f_3$  and  $t^\pm = 1/\sqrt{2}$  reflects the universal coupling to the "charged current".

In the same approximation Sudakov logarithms from real emission processes can be derived on the basis of universal splitting functions. We thus start in this case from the cross section differential in  $z = 1 + M^2/s - 2E_W/\sqrt{s}$ , where  $E_W$  is the energy of the emitted gauge boson in the center of mass frame. The collinear logarithms associated with the outgoing fermion  $f_3$  now become

$$\frac{d\sigma^{(R, col \ f_3)}}{dz} \simeq \frac{\alpha}{2\pi} \left[ \frac{1+z^2}{1-z} \ln \frac{(1-z)^2 s}{M^2} \right] T_{f_3 f'}^A \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f' \bar{f}_4} T_{f_3 f''}^A \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f'' \bar{f}_4} \sigma_B^0. \quad (16)$$

Integrating this contribution in the kinematic limits  $0 \leq z \leq (1 - M/\sqrt{s})^2$  and disentangling again the contributions from  $W^3$  and  $W^\pm$  emission, yields

$$\sigma^{(R, col \ f_3)} \simeq \frac{\alpha}{4\pi} \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] \left( (t_{f_3}^3)^2 \sigma_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} + (t^\pm)^2 \sigma_B^{f_1 \bar{f}_2 \rightarrow f_3^\pm \bar{f}_4} \right), \quad (17)$$

where  $f_3^\pm$  collectively denotes the isospin conjugate of the fermion  $f_3$ , i.e.  $u^- = d$  and  $d^+ = u$ . Together with (15) we see that the Sudakov logarithms from  $W^3$  exchange cancel between virtual and real corrections. This is, however, different for  $W^\pm$  exchange since the individual contributions factorize to different Born cross sections.

Let us briefly comment on the situation when the considered fermion is in the initial state. The differential cross section contains in this case an additional factor  $1/z$ , since the center of mass energy of the hard subprocess has been lowered by the emission process. One may further proceed along the lines of our explicit calculation in (10), which again yields a Sudakov factor as in (17) and a remnant collinear logarithm in the plus-distribution which is left uncanceled.

Soft gauge boson radiation induces further single logarithms. In contrast to the collinear logarithms considered so far, the soft logarithms are angular dependent and stem from interference effects. We thus have to consider gauge boson exchange between pairs of particles. The soft logarithms can be derived in the eikonal approximation, which for an exchange between the incoming fermion  $f_1$  and the outgoing fermion  $f_3$  yields

$$\frac{d\sigma^{(V, soft \ f_1 f_3)}}{dt_{13}} \simeq -\frac{\alpha}{2\pi} \ln^2 \frac{|t_{13}|}{M^2} \frac{d\sigma_B^0}{dt_{13}} T_{f' f_1}^A T_{f_3 f''}^A \mathcal{C}_B^{f' \bar{f}_2 \rightarrow f'' \bar{f}_4} \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4}, \quad (18)$$

where  $t_{13} = (p_{f_3} - p_{f_1})^2$ . We next reshuffle the logarithm according to

$$\ln^2 \frac{|t_{13}|}{M^2} = \ln^2 \frac{s}{M^2} + 2 \ln \frac{|t_{13}|}{s} \ln \frac{s}{M^2} + \ln^2 \frac{|t_{13}|}{s}, \quad (19)$$

and discard the double Sudakov logarithm since it originates from the soft-collinear momentum region that we already accounted for in the collinear approximation. The single angular-dependent logarithm in the second term then leads to

$$\sigma^{(V,soft\ f_1 f_3)} \simeq -\frac{\alpha}{\pi} N_{13} \ln \frac{s}{M^2} \left( t_{f_1}^3 t_{f_3}^3 \sigma_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} + (t^\pm)^2 \mathcal{C}_B^{f_1^\pm \bar{f}_2 \rightarrow f_3^\pm \bar{f}_4} \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} \sigma_B^0 \right), \quad (20)$$

where the prefactor  $N_{13}$  encodes the angular integration,

$$N_{13} = \int dt_{13} \ln \frac{|t_{13}|}{s} \frac{d\sigma_B^0/dt_{13}}{\sigma_B^0}, \quad (21)$$

which, despite our assumption  $s, |t|, |u| \gg M^2$ , can be performed over all angles in the logarithmic approximation. Let us add that the single soft logarithms are absent if we pair two particles that are both in the initial or final state, since  $\ln^2 |s_{12}|/M^2 = \ln^2 |s_{34}|/M^2 = \ln^2 s/M^2$  for the four-fermion process. The eikonal approximation leads, moreover, to additional minus signs if we exchange incoming with outgoing particles and fermions with antifermions.

The corresponding logarithms from real emission processes can be derived similarly within the eikonal approximation. For the same exchange between the incoming fermion  $f_1$  and the outgoing fermion  $f_3$  we obtain

$$\frac{d^2 \sigma^{(R,soft\ f_1 f_3)}}{dt_{13}} \simeq \frac{\alpha}{2\pi} \ln^2 \frac{|t_{13}|}{M^2} \frac{d\sigma_B^0}{dt_{13}} T_{f' f_1}^A \mathcal{C}_B^{f' \bar{f}_2 \rightarrow f_3 \bar{f}_4} T_{f_3 f''}^A \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f'' \bar{f}_4}. \quad (22)$$

Proceeding as before with (19) and extracting the contribution that encompasses the single soft logarithm, we get

$$\sigma^{(R,soft\ f_1 f_3)} \simeq \frac{\alpha}{\pi} N_{13} \ln \frac{s}{M^2} \left( t_{f_1}^3 t_{f_3}^3 \sigma_B^{f_1 \bar{f}_2 \rightarrow f_3 \bar{f}_4} + (t^\pm)^2 \mathcal{C}_B^{f_1^\pm \bar{f}_2 \rightarrow f_3 \bar{f}_4} \mathcal{C}_B^{f_1 \bar{f}_2 \rightarrow f_3^\mp \bar{f}_4} \sigma_B^0 \right). \quad (23)$$

Together with (20) we again see that the  $W^3$  contribution cancels between virtual and real corrections, while the  $W^\pm$  contribution does not due to the modified group structure.

In phenomenological applications one is often interested in observables that are *partly inclusive* in the non-abelian charges (e.g. in processes with light quarks in the final state). Let us therefore briefly address the cross sections  $\sigma_{u\bar{u}}$  and  $\sigma_{u\bar{d}}$ , where the isospin charges of the initial state particles have been fixed while the final state is considered to be inclusive (for the neutral current we thus sum, for instance, over  $u'\bar{u}'W^3$ ,  $u'\bar{d}'W^-$ , etc. where  $u'/d'$  refer to a different isospin doublet than  $u/d$ ). As the BN violations from the final state particles are washed out for these observables, the sum of virtual and real corrections is free from angular-dependent logarithms. We thus obtain a particularly simple result [32],

$$\begin{aligned} \Delta\sigma_{u\bar{u}} &= \sigma_{u\bar{u}}^{(V)} + \sigma_{u\bar{u}}^{(R)} \simeq \frac{\alpha}{4\pi} \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] (t^\pm)^2 \left( \sigma_{u\bar{d}}^B + \sigma_{d\bar{u}}^B - 2\sigma_{u\bar{u}}^B \right), \\ \Delta\sigma_{u\bar{d}} &= \sigma_{u\bar{d}}^{(V)} + \sigma_{u\bar{d}}^{(R)} \simeq \frac{\alpha}{4\pi} \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] (t^\pm)^2 \left( \sigma_{u\bar{u}}^B + \sigma_{d\bar{d}}^B - 2\sigma_{u\bar{d}}^B \right), \end{aligned} \quad (24)$$

where we suppressed the logarithms from initial state radiation that are not supposed to cancel (plus-distributions). Given the Born relations  $\sigma_{u\bar{d}}^B = \sigma_{d\bar{u}}^B = 2\sigma_{u\bar{u}}^B = 2\sigma_{d\bar{d}}^B$ , we get

$$\Delta\sigma_{u\bar{u}} = -\Delta\sigma_{u\bar{d}} \simeq \frac{\alpha}{4\pi} \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] (t^\pm)^2 \sigma_{u\bar{d}}^B. \quad (25)$$

On the basis of the BN violations, we thus expect an overcompensation of the virtual corrections for the inclusive neutral current process, while the real radiation is expected to be less important for the charged current.

### 3.2 NLO calculation with phase space cuts

Let us now investigate the numerical impact of the BN violations and their interplay with the phase space cuts. In contrast to the considerations from the previous section, we now consider the full NLO calculation accounting for double and single Sudakov logarithms as well as constant terms. Power-suppressed terms of  $\mathcal{O}(M^2/s)$ , on the other hand, will be neglected.

In this approximation the one-loop virtual corrections to the  $s$ -channel four-fermion process  $f_1\bar{f}_2 \rightarrow f_3\bar{f}_4$  become

$$\begin{aligned} \sigma^{(V)} \simeq \frac{\alpha}{4\pi} \left\{ -4C_F \left[ \ln^2 \frac{s}{M^2} - 3 \ln \frac{s}{M^2} \right] + \frac{13}{3}C_A \ln \frac{s}{M^2} + \left( \frac{4\pi^2}{3} - 14 \right) C_F \right. \\ \left. + \left( \frac{259}{18} - 2\pi^2 \right) C_A - \frac{40}{9}T_F n_f - \frac{8}{9}n_s \right\} \sigma_B^{f_1\bar{f}_2 \rightarrow f_3\bar{f}_4}, \end{aligned} \quad (26)$$

which implies the same *relative* correction for charged and neutral current processes. The Sudakov logarithms in (26) have a simple interpretation in terms of our formal analysis from the previous section: the Sudakov factors  $\sim C_F$  stem from the collinear approximation (15), while the soft logarithms  $\sim C_A$  result from the various pairings (20) of external particles<sup>3</sup>.

In the next step we include a certain amount of (unobservable) real gauge boson radiation according to our scenarios from Figure 1. In view of the phenomenological applications from Section 4, we will concentrate on the semi-inclusive cross sections  $\sigma_{u\bar{u}}$  and  $\sigma_{u\bar{d}}$  that we introduced at the end of the previous section. For both sets of cuts the relative NLO corrections are shown in Figure 3. First of all we note that the Sudakov suppression is somewhat less pronounced in the non-abelian toy theory due to the prefactor  $C_F = 3/4$  multiplying the Sudakov factors and the impact of the soft logarithms which happen to contribute with opposite sign. Still, the one-loop virtual corrections induce a 10-20% suppression in the TeV regime. Comparing left and right plots, we recognize the qualitative difference between the two scenarios that we worked out in detail for the abelian theory<sup>4</sup>. The abelian picture is, moreover, significantly modified by the

<sup>3</sup>The soft logarithms were absent in the abelian theory, cf. (7), since the sum of all pairings led to an exact cancellation in this case.

<sup>4</sup>Closer inspection reveals that the dependence on the cutoff  $z_c$ , cf. (8) and (11), is also present in the non-abelian theory, while the soft logarithms only slightly modify the angular dependence  $g(\theta_{Ib}^c, \theta_{Fb}^c)$  in (12).

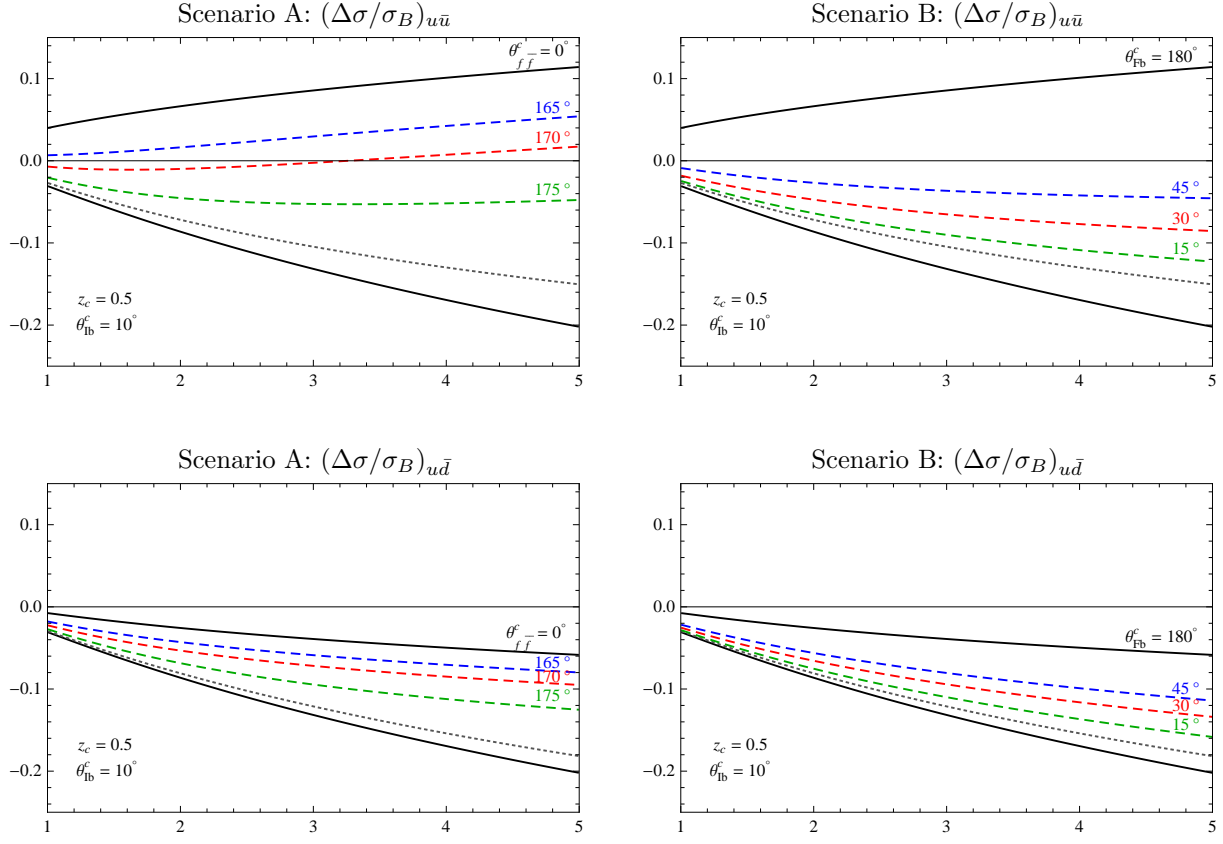


Figure 3: *Relative NLO corrections to the four-fermion process in a spontaneously broken  $SU(2)$  theory with neutral (upper plots) and charged (lower plots) initial states (notation and numerical input values from Figure 2).*

BN violations. This is in particular true, when soft and collinear radiation is allowed by the phase space cuts and the compensation is further favoured by the BN violations (upper left plot). Here even for a tight cut like  $\theta_{f\bar{f}}^c = 175^\circ$  almost complete compensation of the Sudakov suppression is observed, while for a more moderate cut as  $\theta_{f\bar{f}}^c = 165^\circ$  we find an overcompensation of the virtual corrections. It is also interesting to compare the upper right plot (neutral initial state, collinear radiation included) with the lower left plot (charged initial state, soft and collinear radiation included), where one reads off that the differences between the two scenarios can, at least to some extent, be washed out by the BN violations.

In total we find that the large negative corrections from virtual gauge boson exchange can be partially compensated if real radiation is included. The details of this compensation mechanism depend on the isospin configuration of initial and final state particles and on the particular phase space cuts that constrain the real radiation.

## 4 Electroweak Sudakov corrections

Having exemplified the basic concepts behind the compensation of virtual Sudakov corrections in spontaneously broken gauge theories, we may now translate our observations to Standard Model processes. Let us emphasize that this discussion will be, necessarily, of qualitative nature. The efficiency of the cuts on gauge boson radiation will depend on the details of the specific process as, for instance, the fermionic initial and final state (leptons or quark jets), the charge of the radiated gauge boson ( $Z$  or  $W$ ) and its decay mode. Restricting the discussion for example to  $e^+e^-$  colliders, it is plausible that  $\mu^+\mu^-$  will constitute a “clean” final state and gauge boson radiation can be rejected. For quark-antiquark final states, on the other hand, collinear energetic gauge bosons decaying hadronically may well be masked by the quark jets. In contrast, it is plausible that leptonically decaying  $Z$  bosons can be separated from the background. Soft gauge bosons emitted under large angles will again lead to different signatures. The efficiency for detection of gauge boson radiation in hadronic collisions will again be different.

With the kinematics at hadron colliders being less constrained as a consequence of the convolution with parton distribution functions, the following discussion will be restricted to the simpler case of electron-positron collisions while hadron collisions will be treated at a later point [36]. Since one may expect that reactions with leptonic final states will be fairly clean and not “contaminated” by gauge boson radiation, we will concentrate on the process  $e^+e^- \rightarrow q\bar{q}$  (where  $q = u, d$ ). This allows us to study both of the aspects that we discussed in the previous sections in a realistic environment: First, the process is affected by BN violations since the isospin charges of the initial state fermions are singled out and, second, soft and/or collinear gauge boson emission may not always be easily resolved in an experimental setup due to the hadronic signature of the process.

### 4.1 Bloch-Nordsieck violations

Let us first address the BN violations of the current process without restrictions on the gauge boson kinematics. From our analysis in Section 3.1 we deduce that  $Z$  boson (and photon) emission is irrelevant in this context. Moreover, as we sum over the quark flavours  $u$  and  $d$  in the final state, the respective BN violations are washed out. We are thus left with the first of the equations in (24).

In the chiral electroweak theory we have, in addition, to specify the helicity structure of the process. For left-handed leptons in the initial and left or right-handed quarks in the final state, we obtain

$$\begin{aligned}\Delta\sigma_{e^-e^+}^{LL} &\simeq \frac{\alpha}{4\pi s_w^2} \left[ \ln^2 \frac{s}{M_W^2} - 3 \ln \frac{s}{M_W^2} \right] \left( \sigma_{e^-\bar{\nu}}^{B,LL} - \sigma_{e^-e^+}^{B,LL} \right), \\ \Delta\sigma_{e^-e^+}^{LR} &\simeq \frac{\alpha}{4\pi s_w^2} \left[ \ln^2 \frac{s}{M_W^2} - 3 \ln \frac{s}{M_W^2} \right] \left( -\sigma_{e^-e^+}^{B,LR} \right),\end{aligned}\tag{27}$$

respectively, where  $s_w^2 = \sin^2 \theta_w \simeq 0.231$  with  $\theta_w$  being the weak mixing angle. As  $\sigma_{e^-\bar{\nu}}^{B,LL} \simeq 1.98 \sigma_{e^-e^+}^{B,LL}$  the purely left-handed component shows an overcompensation of the virtual

corrections, similar to the neutral current process from the  $SU(2)$  theory in Section 3. The situation is, however, reversed for right-handed quarks since the corresponding charged current process is forbidden at Born level,  $\sigma_{e^- \bar{\nu}}^{B,LR} = 0$ . The BN violations are, moreover, absent for right-handed leptons in the initial state,

$$\Delta\sigma_{e^- e^+}^{RL} = \Delta\sigma_{e^- e^+}^{RR} \simeq 0. \quad (28)$$

The cancellation of the logarithmic  $W$ -corrections for left-handed quarks is a consequence of the fact that we sum over quark flavours in the final state, while the BN violations are of course absent for the purely abelian right-handed component.

We thus see that the chiral coupling of the  $W$ -bosons induces distinct patterns of BN violations that depend on the helicities of the external particles. In the present case this leads, in particular, to interesting effects for polarized lepton beams. For unpolarized beams, on the other hand, the purely left-handed component is expected to dominate the pattern since  $\sigma_{e^- e^+}^{B,LL} \simeq 2.52 \sigma_{e^- e^+}^{B,RR} \simeq 10.1 \sigma_{e^- e^+}^{B,LR} \simeq 25.2 \sigma_{e^- e^+}^{B,RL}$  at Born level.

## 4.2 Numerical analysis

Let us first inspect the virtual corrections to the current process in some detail. Neglecting power-suppressed terms of  $\mathcal{O}(M_{W,Z}^2/s)$  and summing over the helicities of incoming and outgoing fermions, the one-loop electroweak corrections for the annihilation into (massless) up- and down-type quarks can be written as

$$\begin{aligned} \sigma_{e^- e^+ \rightarrow u \bar{u}}^{(V)} &\simeq \frac{\alpha}{4\pi s_w^2} \left\{ -1.28 \left[ \ln^2 \frac{s}{M_W^2} - 3 \ln \frac{s}{M_W^2} \right] + 1.43 \ln \frac{s}{M_W^2} \right. \\ &\quad \left. - 0.39 \left[ \ln^2 \frac{s}{M_Z^2} - 3 \ln \frac{s}{M_Z^2} \right] - 1.12 \ln \frac{s}{M_Z^2} - 8.36 \right\} \sigma_{e^- e^+ \rightarrow u \bar{u}}^B, \\ \sigma_{e^- e^+ \rightarrow d \bar{d}}^{(V)} &\simeq \frac{\alpha}{4\pi s_w^2} \left\{ -1.62 \left[ \ln^2 \frac{s}{M_W^2} - 3 \ln \frac{s}{M_W^2} \right] + 12.57 \ln \frac{s}{M_W^2} \right. \\ &\quad \left. - 0.56 \left[ \ln^2 \frac{s}{M_Z^2} - 3 \ln \frac{s}{M_Z^2} \right] + 1.48 \ln \frac{s}{M_Z^2} - 34.02 \right\} \sigma_{e^- e^+ \rightarrow d \bar{d}}^B, \end{aligned} \quad (29)$$

where we distinguished between Sudakov factors, which encode the collinear logarithms, and single soft logarithms from  $W$ - and  $Z$ -boson exchange. Note that the soft logarithms come in the latter case with a large positive coefficient, which significantly reduces the Sudakov suppression in the few TeV region. The relative corrections to the inclusive process  $e^+ e^- \rightarrow q \bar{q}$  amount, for instance, to  $-2.7\%$  ( $-6.6\%$ ) at  $\sqrt{s} = 1$  TeV (2 TeV), respectively.

One comment is in order concerning our treatment of QED divergences. As our prior interest are "genuine" electroweak effects from  $W$ - and  $Z$ -boson emission, we will disregard Sudakov effects of pure QED nature. In other words we do *not* include real photon emission in our analysis, but rather subtract the QED divergences, which we



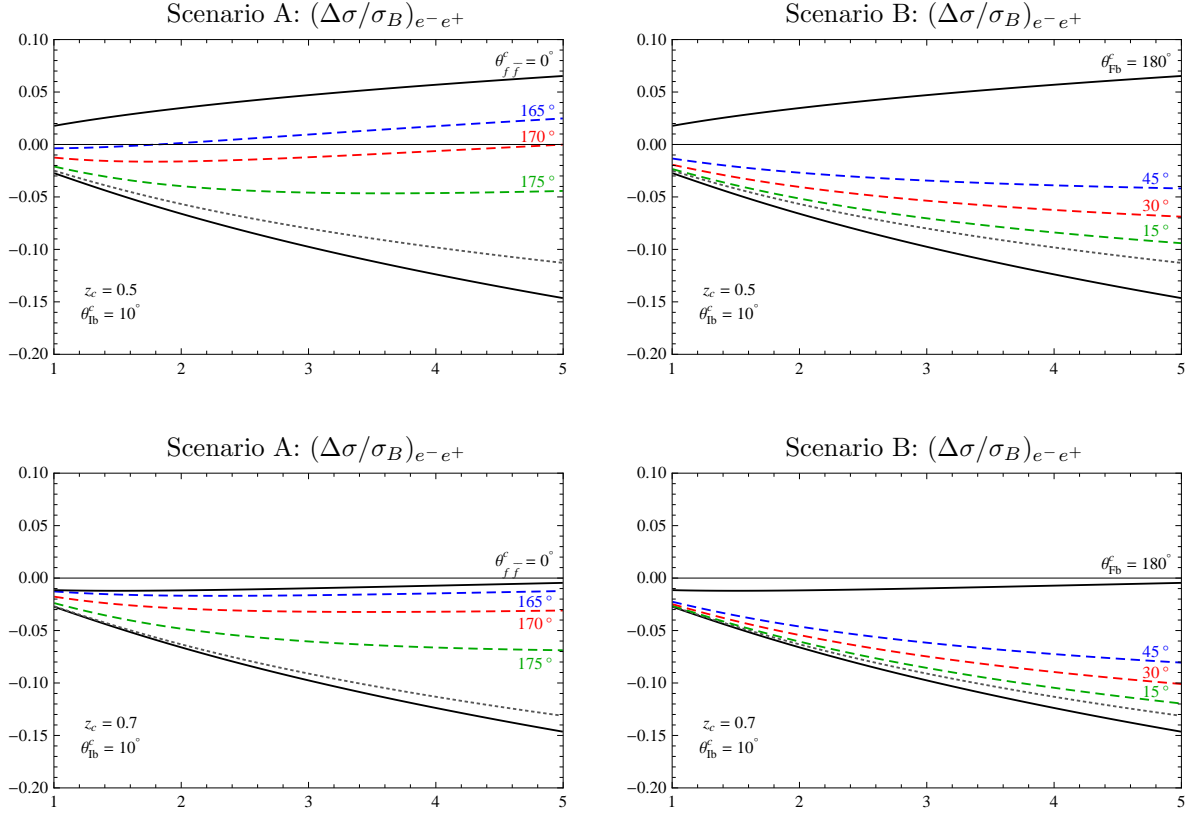


Figure 4: *Relative NLO electroweak corrections to  $e^+e^- \rightarrow q\bar{q}$  as a function of the center of mass energy in TeV. In each plot the lower solid line represents the virtual correction (with  $\alpha = 1/128$  and  $s_w^2 = 0.231$ ) and the dashed lines refer to the sum  $\Delta\sigma = \sigma^{(V)} + \sigma^{(R)}$  with different restrictions on the real emission process (according to the scenarios from Figure 1). The individual dashed lines (green/red/blue, from bottom to top in each plot) refer to  $\theta_{f\bar{f}}^c = 175^\circ/170^\circ/165^\circ$  (left) and  $\theta_{Fb}^c = 15^\circ/30^\circ/45^\circ$  (right). We further set  $z_c = 0.5/0.7$  in the upper/lower plots and  $\theta_{fb}^c = 10^\circ$ . The dotted curves indicate the contribution from initial state radiation (corresponding to  $\theta_{f\bar{f}}^c = 180^\circ$  and  $\theta_{Fb}^c = 0^\circ$ , respectively).*

regularized with a photon mass, from the virtual corrections (they have already been omitted in (29)). In order to obtain a physical cross section, our results thus have to be supplemented by a standard QED correction factor that depends on fermion masses and on specific cuts that constrain the soft photon emission, but is independent of  $M_{W,Z}$ . For the process under consideration this is a gauge invariant separation.

Let us now turn to real  $W$ - and  $Z$ -boson radiation. Focusing again on the process with unpolarized leptons in the initial state and summing over the quark species and polarizations in the final state, we illustrate the size of the BN violations in the upper plots from Figure 4 (adopting the same conventions as in Figure 3). As the process is dominated by the purely left-handed component, we essentially recover the pattern of the neutral current process from the  $SU(2)$  theory, cf. the upper plots from Figure 3. Our default choice of phase space cuts ( $z_c = Q_c^2/s = 0.5$ ,  $\theta_{fb}^c = 10^\circ$ ) may, however, not be quite realistic for the considered process. As an alternative we therefore show the result

for a more restrictive cut on initial state radiation ( $z_c = 0.7$ ,  $\theta_{Ib}^c = 10^\circ$ ) in the lower plots from Figure 4. In other words we demand that the quark-antiquark pair (or rather their associated jets) carries at least about 84% of the beam energy and we assume that  $W$ - and  $Z$ -bosons, that are emitted into the extreme forward direction, cannot be resolved for angles  $\theta_{Ib} \leq 10^\circ$  (which corresponds to a pseudorapidity cut  $|\eta| \geq 2.4$ ). We may then investigate the impact from soft and (final state) collinear  $W$ - and  $Z$ -boson radiation by varying the parameters  $\theta_{f\bar{f}}^c$  and  $\theta_{Fb}^c$ .

From the lower plots in Figure 4 we read off that real gauge boson radiation becomes numerically relevant in the few TeV region only if some fraction of collinear *and* soft  $W$ - and  $Z$ -bosons escapes experimental detection (Scenario A). For reasonable values of phase space cuts as  $\theta_{Fb}^c = 15^\circ$  the Sudakov suppression is, for instance, only marginally reduced in Scenario B from  $-2.7\%$  ( $-6.6\%$ ) to  $-2.6\%$  ( $-6.1\%$ ) at  $\sqrt{s} = 1$  TeV (2 TeV). In contrast to this the impact from real radiation is much more pronounced if some soft non-collinear radiation is accounted for. In Scenario A the Sudakov suppression is, for instance, reduced to  $-1.8\%$  ( $-2.9\%$ ) for  $\theta_{f\bar{f}}^c = 170^\circ$ .

## 5 Conclusions

The purpose of our work was to investigate to which extent real gauge boson radiation can compensate the characteristic negative virtual corrections that arise in high-energy reactions. As the latter are driven by Sudakov logarithms, the compensation mechanism depends obviously on the amount of soft and collinear radiation that is allowed by the phase space cuts. A second interesting element in this context is the mismatch of logarithmically enhanced terms in spontaneously broken gauge theories, a phenomenon called Bloch-Nordsieck violations.

In order to address these issues separately, we subsequently studied a massive abelian gauge theory, a spontaneously broken  $SU(2)$  theory and the electroweak Standard Model. We derived analytical results for some exemplary (and idealized) cuts, which facilitate the qualitative understanding of the compensation mechanism. In our numerical analysis we found remarkable differences for cuts which cover all of the singular regions (collinear *and* soft) and those that include them only partially (collinear *or* soft).

The factorization of soft and collinear singularities can be exploited to compute the Bloch-Nordsieck violations for inclusive cross sections on a process-independent basis. Depending on the non-abelian charges of the external particles, the Bloch-Nordsieck violations can lead to a partial cancellation or to an overcompensation of the virtual corrections. We argued that this can to some extent wash out the qualitative differences of the phase space cuts.

We, in particular, tried to understand to which extent electroweak Sudakov corrections are affected by these issues. To this end we discussed the case of electron-positron annihilation into  $u\bar{u}$  and  $d\bar{d}$  quarks in more detail. We performed an explicit NLO calculation and investigated the impact from unobservable  $W$ - and  $Z$ -boson radiation. While the Sudakov suppression is not particularly pronounced for this specific process, it allowed

us to study the compensation mechanism in a realistic environment. We found that real radiation becomes numerically relevant for this process only if a fraction of collinear *and* soft  $W$ - and  $Z$ -bosons escapes experimental detection.

From the phenomenological point of view real  $W$ - and  $Z$ -boson radiation is certainly more important for hadron collider processes. Current hadron colliders are on the eve of probing the multi-TeV region in which the Sudakov effects become more pronounced. The hadronic environment makes, moreover, the discrimination of real gauge boson radiation much more challenging. Typical observables at hadron colliders are actually largely inclusive; the Drell-Yan process allows, for instance, for an arbitrary number of  $W$ - and  $Z$ -bosons decaying hadronically. We plan to extend the presented analysis to hadron collider processes in a future publication [36].

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